(Chapter 16)

100 points NAME:__________________

Be sure to show work neatly and follow instructions carefully. Name any theorems used

(1) Given the vector field $\vec{F}$(x,y) = (4x + y, x + 2y) and the curve C: $y = 1 - x^2$ from (0,1) to (1,0),

a) Compute the line integral of $\vec{F}$ along C directly, using the definition.

b) Is $\vec{F}$ a gradient field? If so find f(x,y) such that $\vec{F} = \nabla f(x,y)$ and use it to verify your answer in part (a).

c) Does Green's theorem apply to this line integral? If so, evaluate the line integral using Green's theorem, if not, why not?

d) For $\vec{F}$ as above but C is now the square with vertices (0,0), (1,0), (1,1), (0,1) oriented counter-clockwise, compute. $\int_C \vec{F} \bullet d\vec{r}$

(2) Given C is the piecewise smooth curve from (4,0) to (4,4) to (0,0) and back to (4,0) find $\int_C y^2\,dx + x^2\,dy$ using two different methods.

(3)) Evaluate $\iint_S (x^2 + y^2 + z^2)\,dS$ where S is the portion of the cylinder $x^2 + y^2 = 9$ in the first octant that lies below the plane z=y.

(4) Given $\vec{F}$ (x,y,z) = xz **i** + 2xy **j** + 3xy **k** , and C is the boundary of the part of the plane 3x + y + z = 3 in the first octant, oriented clockwise as viewed from above, evaluate $\int_C \vec{F} \bullet d\vec{r}$ TWO ways, (a) directly and (b) using another method. (DETAILS)

(5) Given: $\vec{F}$x,y,z) = < y, x , z > and S is the closed surface formed by the paraboloid $z = x^2 + y^2$ and the plane z = 1 and is oriented by outward unit normals, evaluate $\iint_S \vec{F} \bullet d\vec{S}$ TWO ways: (a) directly, and (b) using another method (DETAILS).